Authors' Reply to Comment by N. Curle

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THE present authors had indicated in their note[1] that other methods of calculating the laminar separation point had been proposed but that it was not their concern to review them there. The method of Stratford as modified by Curle and Skan,[2, 3] for incompressible flow, certainly is noteworthy for its simplicity and its agreement with various exact solutions. This, however, does not contradict the fact that the method of Morduchow and Clarke[1, 4] is, first of all, also relatively simple, involving at most a numerical quadrature. (Indeed, the quadrature takes into account, in a way, the previous history of the boundary layer.) Moreover, the formula of Morduchow and Clarke holds both for incompressible flow and for compressible flow with zero heat transfer, and the calculations are virtually as simple for the latter case as for the former. This is evidenced by the ease with which the cases in Ref. 1 were calculated by this means.

With regard to accuracy, considering first incompressible flow, it hardly can be denied that the agreement of the formula of Morduchow and Clarke with essentially exact solutions is very good[1] and certainly is sufficient for practical purposes. For example, the values of ξ_s for the case $u_1/u_\infty = 1 - \xi^n$ are everywhere within 2.5% of Tani's values.[1, 3] Such agreement is especially noteworthy since the method of Ref. 4 is entirely self-contained and makes no use of any known exact solutions for either incompressible or compressible flow.

As regards compressible flow, Curle states that the present authors' agreement with Stewartson's values of ξ_s vs M_∞ for $u_1/u_\infty = 1 - \xi$ simply means that both sets of values are in "about 30 or 40%" error at $M_\infty = 4$. He bases this on the value $\xi_s = 0.045$ ($\pm 10\%$) for $M_\infty = 4$, reported by him[5] and obtained by Mathematics Division, National Physical Laboratories. No details at all are given of how this computation, which is far from routine and involves, for example, a singularity at the separation point (cf., the machine calculation for the incompressible case[6]), was carried out. There have been some other purportedly fairly accurate calculations of this case which do not agree with the National Physical Laboratories value. These results, which are discussed below and shown in Table 1, indicate that it appears rather premature at present to infer that the present authors' results have such a large quantitative error at $M_\infty \geqslant 4$.

Stewartson's calculations[7] are based on the application of his well-known transformation to obtain a corresponding incompressible flow and then the calculation of the latter flow by means of Howarth's polygonal method.[8] Tani[9] used a two-parameter method based on the momentum and energy integral boundary layer equations. His results are seen to be in good agreement with the present authors' (or Stewartson's) for all Mach numbers calculated, especially if corrections for Prandtl number and viscosity are considered. Tani's results are based on the Sutherland viscosity-temperature relation and on a Prandtl number Pr of 0.72. According, for example, to Gadd,[10] the effect of changing Pr from 1 to 0.72 is to move the separation point downstream by about 7 or 8% at $M_\infty = 4$, with a similar effect at other Mach numbers. If this effect is considered at $M_\infty = 4$, it is seen that Tani's results at $M_\infty = 4$ (and at other M_∞) will be brought still closer to the present authors'; a correction[10] for nonlinearity of the $\mu - T$ relation probably also would move Tani's values still closer. As previously indicated,[1] Wrage[11] has recently calculated this case by extending Görtler's series method to compressible flows, in conjunction with a finite-difference method and an electronic computer. Table 1 shows that Wrage's results are substantially similar to the present authors'; the largest deviation appears to be at $M_\infty = 6$ (viz., about 13%), but, interestingly, Wrage's results come still closer at $M_\infty = 10$. An application of Stewartson's transformation to Thwaites' method[12] leads to results[10] quite similar to the present authors'. (A perusal of the literature has indicated that there have been a variety of still other calculations of ξ_s for the problem treated here, and some of them deviate considerably at various Mach numbers from the results in Table 1. However, in each such case the method used was admittedly very approximate and gave appreciable error at $M_\infty = 0$. Moreover, the results of these methods for various M_∞ deviated considerably from one another, without the consistent agreement for all M_∞ shown by the various results in Table 1.)

Table 1 Separation point for $u_1/u_\infty = 1 - \xi$, $\mu \sim T$, and $Pr = 1$ (except Tani), zero heat transfer

M_∞	0	1	3	4	6	10
ξ_s (Morduchow-Clarke)	0.122	0.113	0.077	0.062	0.042	0.023
ξ_s (Stewartson)	0.120	0.110	0.077	0.062	0.044	0.024
ξ_s (Tani; $T_\infty = 72°$R; $Pr = 0.72$)	0.120	0.111	0.081	0.066	0.048	...
ξ_s (Tani; $T_\infty = 648°$R; $Pr = 0.72$)	0.120	0.112	0.085	0.072	0.057	...
ξ_s (Wrage)	0.125	0.112	0.072	0.056	0.037	0.021
ξ_s (Thwaites-Stewartson)	0.120	0.111	...	0.060	0.043	0.023

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Received by IAS December 6, 1962.
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May 2	AIAA-ASMA Manned Space Laboratory Conference[3]	Los Angeles, Calif.	Past
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May 20–22	AIAA-IEEE-ISA National Telemetering Conference	Albuquerque, New Mex.	Past
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